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Classical Methods of Structural Analysis

Louis F. Geschwindner

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Session Description

20.2 Strain Energy and Real Work
June 17, 2019

This lesson will begin with developing a definition for work and the principles of real work and virtual work. The lesson will continue with formulating the equations for strain energy. Calculating deflections by real work will be discussed and a systematic notation for deflections and the Law of Reciprocal Deflections will be developed.

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Learning Objectives:

- Determine deflections by virtual work.
- Formulate the equations for strain energy.
- Calculate deflections by real work.
- Develop a systematic notation for deflections.

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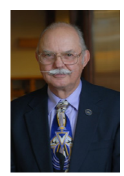
Classical Methods of Structural Analysis

Session 2: Strain Energy and Real Work
June 17, 2019



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Classical Methods of Structural Analysis:
How we did it before computers

Night School 20
Lesson 2
Strain Energy and Real Work

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Lesson 2
Strain Energy and Real Work

- Develop the definition for work.
- Use the principle of virtual displacements to determine reactions.
- Formulate the equations for strain energy.
- Calculate deflections by real work.
- Develop a systematic notation for deflections.
- Develop the Law of Reciprocal Deflections.
- Establish the need for determining deflections by virtual work.

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Work

- A force is said to do work when it acts on a body, and there is a displacement of the point of application in the direction of the force.

The kids are doing "work" when they lift the steel beam off the ground. But, as they move the beam horizontally to another location, they are not doing any "work." When they put the beam down, they are doing negative work.

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Work

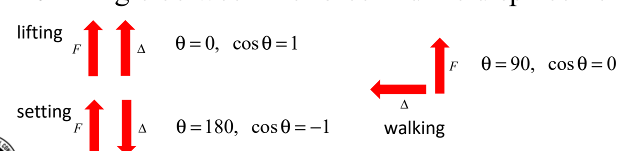
$$W = F\Delta\cos\theta$$

$W =$ total work done

$F =$ force doing the work

$\Delta =$ displacement the force moves through

$\theta =$ angle between the force and the displacement



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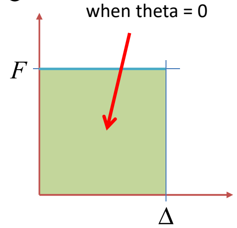
Work

$$W = F\Delta\cos\theta$$



$W =$ total work done

$F =$ force doing the work

$\Delta =$ displacement of the force

$\theta =$ angle between the force and the displacement

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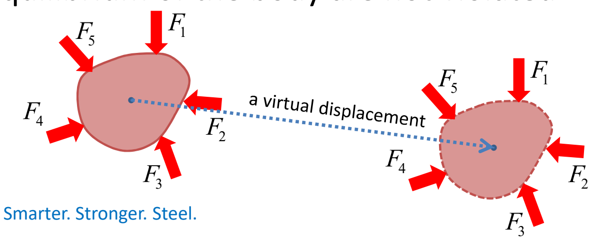
Virtual Displacements

- A *virtual displacement* denotes a hypothetical displacement, either finite or infinitesimal, of a point or system of points on a rigid body in equilibrium such that the equations of equilibrium of the body are not violated



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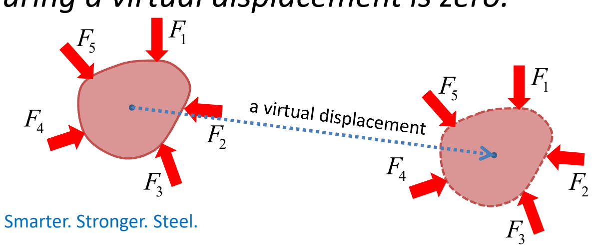
Virtual Displacements

- Principle of Virtual Displacements: *given a rigid body held in equilibrium by a system of forces and/or couples, the total virtual work done by this system of forces and/or couples during a virtual displacement is zero.*



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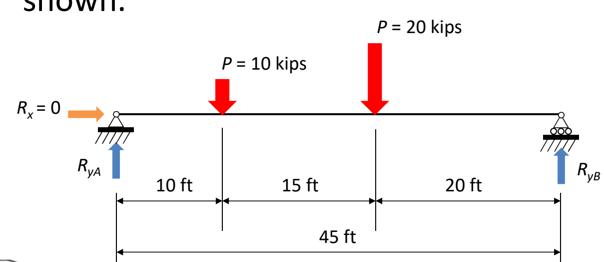
Virtual Displacements

- Using the principle of virtual displacements, determine the reaction R_{yB} on the beam shown.



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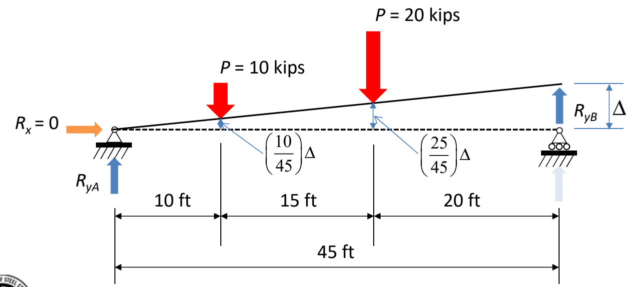
Virtual Displacements

- Displace the beam, as a rigid body, through a displacement at end B.



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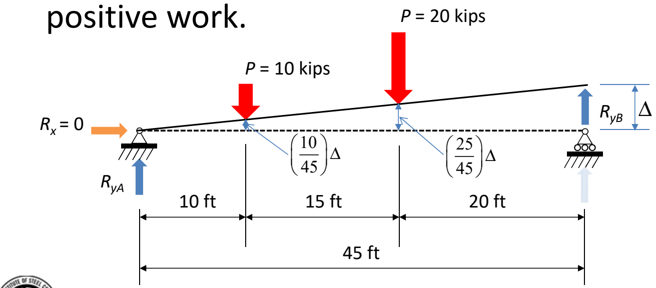
Virtual Displacements

- Note that the 10 kip and 20 kip loads do negative work ($\theta = 180^\circ$). The reaction at B does positive work.



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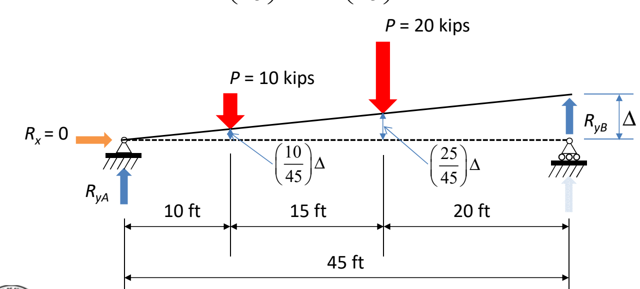
Virtual Displacements

- Thus

$$W_{VD} = R_{yB}\Delta - 10\left(\frac{10}{45}\right)\Delta - 20\left(\frac{25}{45}\right)\Delta = 0, \qquad R_{yB} = 13.33 \text{ kips}$$



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Real Work

- In 1833, Clapeyron established that the external work done by the loads deflecting a structure and the internal work done (strain energy) were equal.
  - When loads are applied to a deformable structure they will do work as the points of application are displaced.
  - If the reactions don't move, then all the external work of the applied loads must be dissipated in straining the structural material.

External Work = Internal Work

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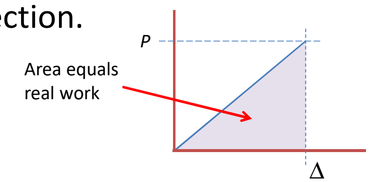
Real Work

- As a load is gradually applied to a structure, the point of application deflects and comes to rest with a total deflection, Δ.
- Since the principle of superposition applies, there is a linear relationship between load and deflection.



The average value of the load acting during the deflection is *P*/2, therefore the external work is

$$W_e = \frac{P\Delta}{2}$$

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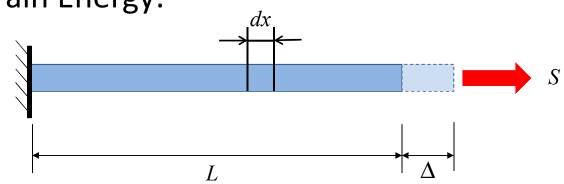
Internal Work

- Axial Strain Energy:



The internal work in the length dx will be equal to the average load times the change in length over the length dx.

$$dW_i = \frac{S}{2}\frac{\Delta}{L}dx$$

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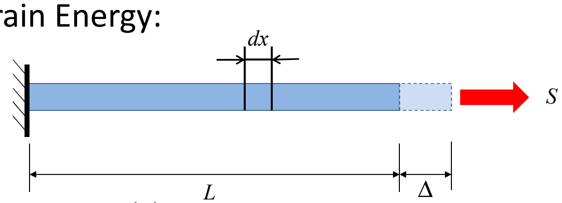
Internal Work

- Axial Strain Energy:



From Hooke's Law $E = \frac{f}{\varepsilon} = \frac{\left(\frac{S}{A}\right)}{\left(\frac{\Delta}{L}\right)}$; which leads to $\Delta = \frac{SL}{AE}$

Thus,

$$dW_i = \frac{S}{2}\frac{\Delta}{L}dx = \frac{S}{2}\frac{SL}{AE}\frac{dx}{L} = \frac{S^2 dx}{2AE}$$

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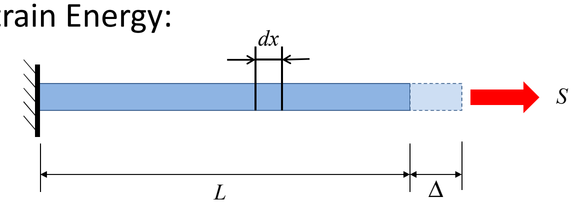
Internal Work

- Axial Strain Energy:



And the total internal work for the entire bar will be

$$W_i = \int_0^L \frac{S^2 dx}{2AE} = \frac{S^2 L}{2AE}$$

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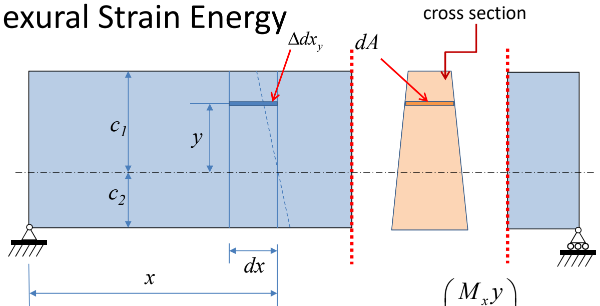
Internal Work

- Flexural Strain Energy



From Hooke's Law, $E = \dfrac{f}{\varepsilon} = \dfrac{\left(\dfrac{M_x y}{I}\right)}{\left(\dfrac{\Delta dx_y}{dx}\right)}$

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Internal Work

- Flexural Strain Energy

From Hooke's Law, the change in length in a fiber y distance from the neutral axis in the length dx is thus,

$$\Delta dx_y = \frac{M_x y}{I}\frac{dx}{E}$$

The stress on the same fiber is

$$f_y = \frac{M_x y}{I}$$

and the force at that location is

$$F_y = f_y dA = \frac{M_x y}{I} dA$$

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Internal Work

- Flexural Strain Energy

The internal strain energy in the length dx of the fiber in question is therefore,

$$\frac{1}{2}(F_y)(\Delta dx_y) = \frac{1}{2}\left(\frac{M_x y dA}{I}\right)\left(\frac{M_x y}{I}\frac{dx}{E}\right) = \frac{1}{2}\left(\frac{M_x y}{I}\right)^2 \frac{dAdx}{E}$$

The total internal strain energy for all fibers in the length dx will be determined by integrating over the area,

$$dW_i = \frac{1}{2}\int_{-c_2}^{c_1}\left(\frac{M_x y}{I}\right)^2 \frac{dAdx}{E} = \frac{1}{2}\frac{M_x^2 dx}{EI^2}\int_{-c_2}^{c_1} y^2 dA = \frac{1}{2}\frac{M_x^2 dx}{EI}$$

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Internal Work

- Flexural Strain Energy

For the full length of the beam we integrate over the length.

$$W_i = \int_0^L \frac{M_x^2 dx}{2EI}$$

Unlike the situation for axial loading, we can not determine this quantity without the integration since we do not know the distribution of moment along the length of the beam. This will come later.

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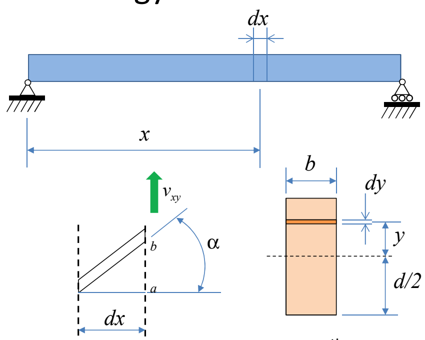
Internal Work

- Shear Strain Energy



cross section

We will first develop this for a rectangular cross section.

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Internal Work

- Shear Strain Energy

At fiber y, the shear stress is v_{xy}. When this stress acts over the area dA, the shear force at y is $v_{xy}\,dA$.

The distance that force moves through is the distance from *a* to *b* which, considering small angle theory, is αdx .

Thus, $$w_i = \frac{1}{2}\left(v_{xy} dA\right)\alpha dx$$

where $dA = bdy$ and $\alpha = \dfrac{v_{xy}}{G}$

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Internal Work

- Shear Strain Energy

Substitution yields

$$w_i = \frac{1}{2}\left(v_{xy} dA\right)\alpha dx = \frac{1}{2}\left(v_{xy} bdy\right)\frac{v_{xy}}{G}dx$$

The shear stress at *y* above the neutral axis is

$$v_{xy} = \frac{V_x Q_y}{Ib}$$

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Internal Work

- Shear Strain Energy

Substituting for v_{xy} and integrating over y for the rectangle yields the work at location x on the beam

$$dW_i = \frac{1.2V_x^2}{2AG}dx$$

The total work for the beam is obtained by integrating over the span. Thus,

$$W_i = \int_0^L \frac{1.2V_x^2}{2AG}dx$$

For a rectangular cross section

$$W_i = K\int_0^L \frac{V_x^2}{2AG}dx$$

For any cross section

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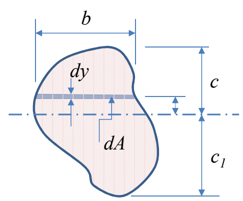
Internal Work

- Shear Strain Energy
  - K in this equation is the shape constant. For a generic shape,



$$K = A\int_{c_1}^{c} \frac{1}{I^2 b}\left[\int_{y}^{c} ybdy\right]^2 dy$$

For a rectangle we already saw that $K = 1.2$.

For a circular cross section, $K = 10/9$.

For W-shape (steel beam) we usually assume that the shear stress distribution is uniform over the web. This results in $K = 1.0$.

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Internal Work

- Torsion Strain Energy

$$W_i = \int_0^L \frac{S^2 dx}{2AE} = \frac{S^2 L}{2AE}$$

Axial

$$W_i = \int_0^L \frac{M_x^2 dx}{2EI}$$

Flexural

$$W_i = K\int_0^L \frac{V_x^2 dx}{2AG}$$

Shearing

By noting the analogy between the three already derived, it can be surmised that for torsion,

$$W_i = \int_0^L \frac{T_x^2 dx}{2JG}$$

Where J = torsional constant. For a shape composed of rectangles,

$$J = \sum bt^3$$

and for W-shapes, it is tabulated in the AISC *Manual*

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Work

- Question to consider
  - Which of the following action does the most work?
    a. Lifting a 5 pound sack 3 feet
    b. Setting a 5 pound sack down from a height of 5 feet
    c. Walking 10 feet carrying a 5 pound sack
    d. A beam that deflects 6 in. when a 40 pound sack is placed on it

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Polling Question

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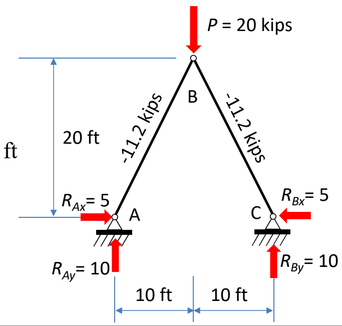
Deflections by Work

- Using the equality of external and internal work, determine the vertical deflection of the top node of the axially loaded structure given.

Members AB and BC

$$A = 4.5 \text{ in}^2$$

$$L = \sqrt{10^2 + 20^2} = 22.4 \text{ ft}$$



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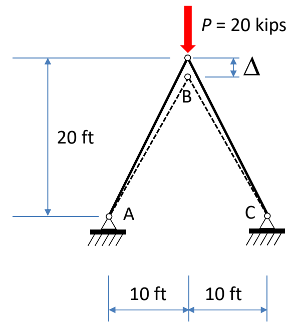
Deflections by Work



External work

$$W_e = \frac{20\Delta}{2} = 10\Delta \text{ kips}$$

Internal work

$$W_i = \sum \frac{S^2 L}{2AE}$$

$$= \frac{(-11.2)^2(22.4)(12)}{2(4.5)(29{,}000)}$$

$$+ \frac{(-11.2)^2(22.4)(12)}{2(4.5)(29{,}000)}$$

$$= 0.258 \text{ kip-in.}$$

Thus,

$$10\Delta = 0.258$$

$$\Delta = 0.0258 \text{ in.}$$

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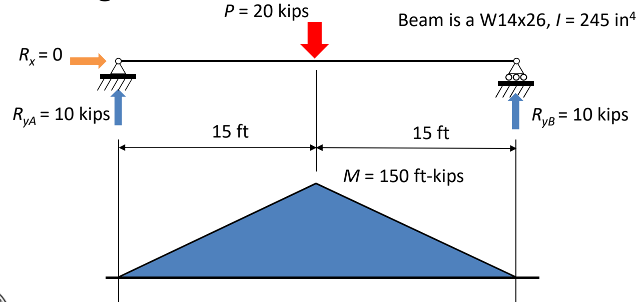
Deflections by Work

- Determine the deflection at mid-span due to bending.



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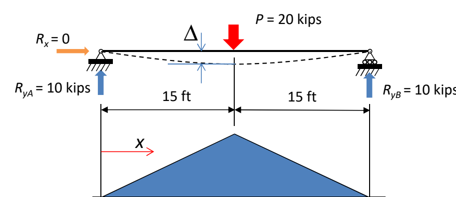
Deflections by Work



External work

$$W_e = \frac{20\Delta}{2} = 10\Delta \text{ kips}$$

To determine the internal strain energy, we must write equations to describe the moment along the span.

From A to mid-span (x = 0 to 15 ft)

$$M_x = 10x$$

From mid-span to B (x = 15 ft to 30 ft)

$$M_x = 10x - 20(x - 15) = 300 - 10x$$

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Deflections by Work

Internal work

$$W_i = \int_0^L \frac{M_x^2 dx}{2EI} = \int_0^{15} \frac{(10x)^2\, dx}{2EI} + \int_{15}^{30} \frac{(300-10x)^2\, dx}{2EI}$$

$$= \frac{100x^3}{6EI}\Bigg|_0^{15} + \frac{90,000x - 6,000\frac{x^2}{2} + 100\frac{x^3}{3}}{2EI}\Bigg|_{15}^{30}$$

$$= \frac{112,500}{EI}\ \text{kip}^2\text{-ft}^3$$

Thus,

$$10\Delta = \frac{112,500(1728)}{(29,000)(245)}$$

and

$$\Delta = 2.74 \text{ in.}$$

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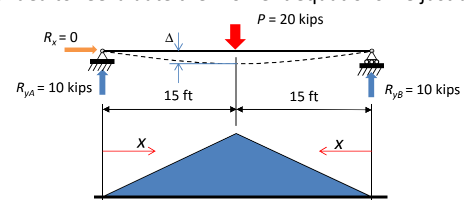
Deflections by Work

- Since the way we write the moment equations will have a significant impact on the complexity of the integrations we must perform, it would be a good idea to reevaluate the moment equations we just used.



Redefine *x* for the right half of the beam as going from B to mid-span. Then the moment equation for that segment goes from 0 to 15 and is identical to that for the left half span.

$$M_x = 10x$$

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Deflections by Work

Internal work

$$W_i = 2\int_0^{15} \frac{(10x)^2\, dx}{2EI} = \frac{100x^3}{3EI}\Bigg|_0^{15} = \frac{112,500}{EI}$$

and from $W_e = W_i$

$$10\Delta = \frac{112,500(1728)}{(29,000)(245)}$$

Thus, as before,

$$\Delta = 2.74 \text{ in.}$$

Note: Always look for ways to define x so that the moment equations are as simple as possible.

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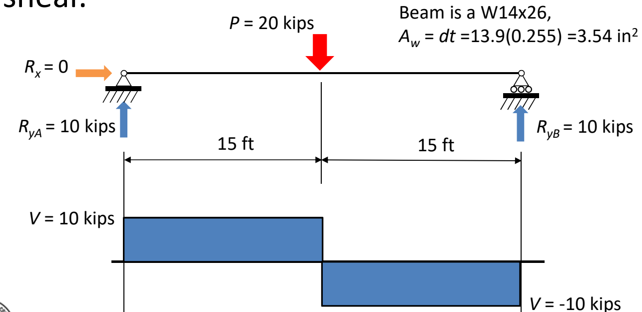
Deflections by Work

- Now consider deflection at mid-span due to shear.

Beam is a W14x26,
$A_w = dt$ =13.9(0.255) =3.54 in^2



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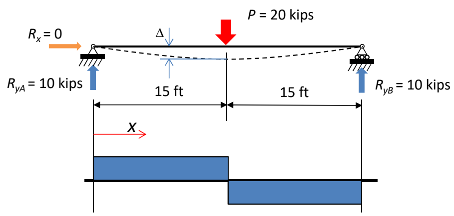
Deflections by Work



External work

$$W_e = \frac{20\Delta}{2} = 10\Delta \text{ kips}$$

To determine the internal strain energy, we must write equations to describe the shear along the span.

From A to mid-span (x = 0 to 15 ft)

$$V_x = 10$$

From mid-span to B (x = 15 ft to 30 ft)

$$V_x = -10$$

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Deflections by Work

Internal work

$$W_i = \int_0^L \frac{V_x^2}{2AG}dx = \int_0^{15} \frac{(10)^2}{2AG}dx + \int_{15}^{30} \frac{(-10)^2}{2AG}dx = \frac{100x}{2AG}\bigg|_0^{15} + \frac{100x}{2AG}\bigg|_{15}^{30} = \frac{1500+3000-1500}{2AG}$$

and from $W_e = W_i$

$$10\Delta \text{ kips} = \frac{3{,}000(12)}{2(3.54)(11{,}200)} = 0.454 \text{ kip-in}$$

Thus,

$$\Delta = 0.0454 \text{ in.}$$

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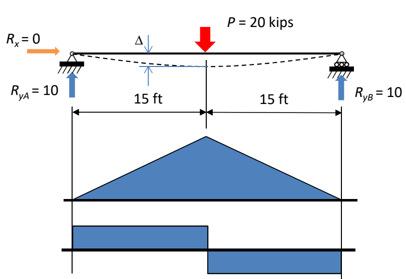
Deflections by Work

- Since AISC 360-16 Section C1 requires that flexural, shear, and axial deformations be considered, (note axial is zero) how would we incorporate shear and flexure?



$$W_e = W_i = \int_0^L \frac{M_x^2}{2EI}dx + \int_0^L \frac{V_x^2}{2AG}dx$$

$$10\Delta = \frac{112{,}500(1728)}{(29{,}000)(245)} + \frac{3{,}000(12)}{2(3.54)(11{,}200)}$$

$$\Delta = 2.74 + 0.0454 = 2.79 \text{ in.}$$

Thus, it is simply the sum of the two contributing components.

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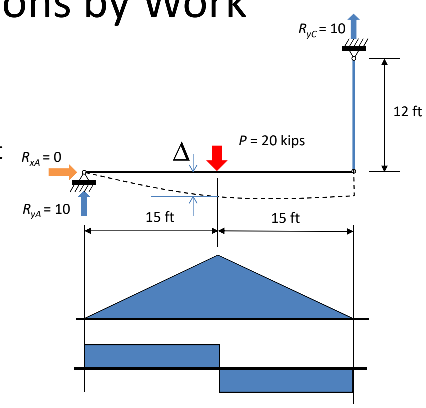
Deflections by Work

- Reconsider this simple beam if the right end is supported by a cable that can stretch.
- Cable:

$$A = 0.5 \text{ in.}^2$$
$$L = 12.0 \text{ ft}$$
$$E = 20{,}000 \text{ ksi}$$



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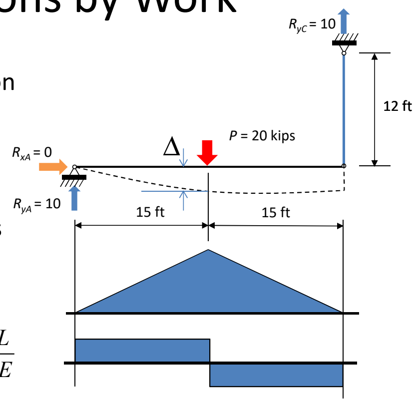
Deflections by Work

- The external work equation has not changed.

$$W_e = \frac{20\Delta}{2}$$

- The internal work changes by the contribution of the cable.

$$W_i = W_{i\ \text{flexure}} + W_{i\ \text{shear}} + \sum \frac{S^2 L}{2AE}$$



Deflections by Work

- For the beam supported by the cable

$$W_e = W_i = \int_0^L \frac{M_x^2}{2EI} dx + \int_0^L \frac{V_x^2}{2AG} dx + \sum \frac{S^2 L}{2AE}$$

$$10\Delta = \frac{112{,}500(1728)}{(29{,}000)(245)} + \frac{3{,}000(12)}{2(3.54)(11{,}200)} + \frac{(10)^2(12)(12)}{2(0.5)(20{,}000)}$$

$$\Delta = 2.74 + 0.0454 + 0.720 = 3.51 \text{ in.}$$

flexural shear axial

Deflections by Work

- For a simple steel beam with a concentrated load at mid-span, determine the relationship between the shear and flexure contributions to deflection.

$$\frac{P\Delta}{2} = 2\int_0^{L/2} \frac{\left(\frac{P}{2}x\right)^2 dx}{2EI} \rightarrow \Delta = \frac{PL^3}{48EI}$$

$$\frac{P\Delta}{2} = 2\int_0^{L/2} \frac{\left(\frac{P}{2}\right)^2}{2AG} dx \rightarrow \Delta = \frac{PL}{4AG}$$

Deflections by Work

- The ratio of shear deflection to bending deflection is:

$$\frac{\Delta_V}{\Delta_M} = \frac{\left(\frac{PL}{4AG}\right)}{\left(\frac{PL^3}{48EI}\right)} = \frac{12EI}{AGL^2}$$

  - Making the following substitutions

$$A = A_w = dt \qquad \frac{E}{G} = 2.5$$

  - And multiplying by

$$\frac{d^2}{d^2}$$

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Deflections by Work

- yields

$$\frac{\Delta_V}{\Delta_M} = \frac{30I}{td^3}\left(\frac{d}{L}\right)^2 = C_2\left(\frac{d}{L}\right)^2$$

It is seen that the relationship between shearing and flexural contributions to deflection is a function of the depth to span ratio squared. Thus, shear is neglected for usual beams since they are relatively shallow and have longer spans.

The variable C_2 can be determined for all W-shapes. It is quite variable and ranges from approximately 24.8 for a W14x90 to 7.80 for a W24x55.

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Deflections by Work

- yields

$$\frac{\Delta_V}{\Delta_M} = \frac{30I}{td^3}\left(\frac{d}{L}\right)^2 = C_2\left(\frac{d}{L}\right)^2$$

A common rule of thumb is that the ideal depth of a steel beam is 1/24 the span. This is also stated as "half the span in inches."

Thus, for the W14x90 $\frac{\Delta_V}{\Delta_M} = 24.8\left(\frac{1}{24}\right)^2 = 0.043$

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Deflections by Work

- Question to consider
  - What deflections can be calculated by the method of work just discussed?
    a. Any deflection
    b. Only deflections on beams
    c. Only deflections under loads
    d. Only the deflection under a single load

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Polling Question

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Notations for Deflections

- For future reference, it will be helpful to have a systematic approach to identifying deflection and rotation components.
- It will also be helpful to take advantage of superposition and the use of unit loads.

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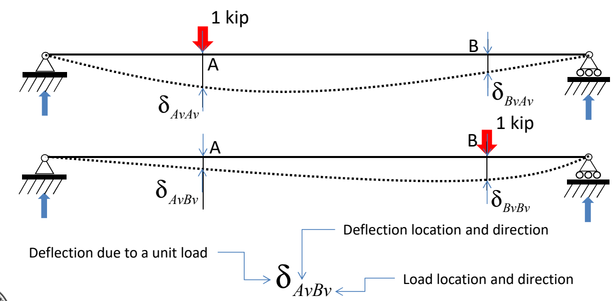
Notations for Deflections

- Consider a structure with points A and B defined and a unit load applied, first at A then at B.



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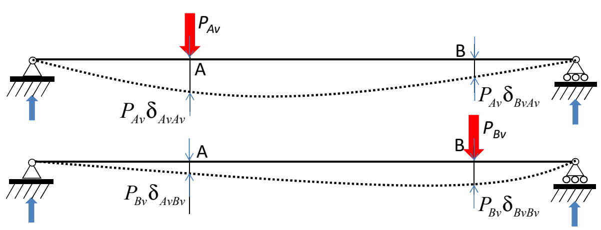
Notations for Deflections

- Apply actual loads independently and identify deflections



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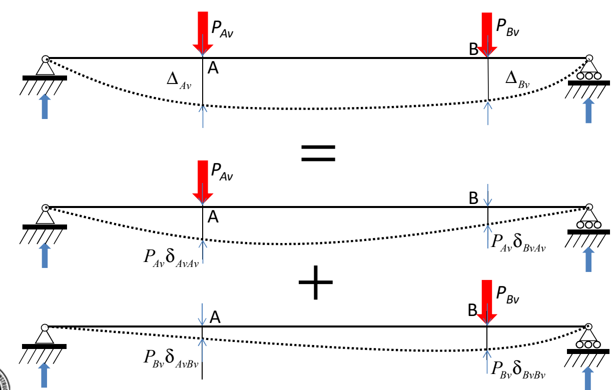
Notations for Deflections



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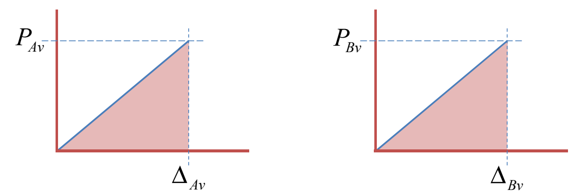
Maxwell's Law of Reciprocal Deflections

- Along with the notation scheme just presented, there is a very useful relationship between deflections that was presented in 1864 by Clerk Maxwell.
- We will develop this relationship using a simple beam where the deflections considered are both vertical.
- This is not a necessary limitation of the method.

Maxwell's Law of Reciprocal Deflections

- Start by applying both loads to the beam we just considered gradually and simultaneously.
- The external work will be

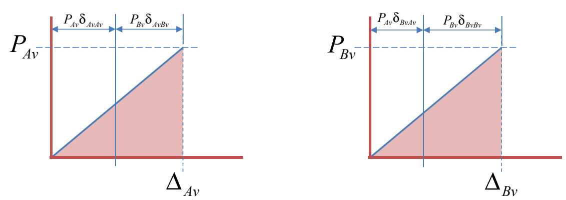
$$W_e = \frac{P_{Av}\Delta_{Av}}{2} + \frac{P_{Bv}\Delta_{Bv}}{2}$$



Maxwell's Law of Reciprocal Deflections

- Which can be written, using superposition

$$W_e = \frac{P_{Av}\Delta_{Av}}{2} + \frac{P_{Bv}\Delta_{Bv}}{2} = \frac{P_{Av}}{2}\left(P_{Av}\delta_{AvAv} + P_{Bv}\delta_{AvBv}\right) + \frac{P_{Bv}}{2}\left(P_{Av}\delta_{BvAv} + P_{Bv}\delta_{BvBv}\right)$$



Maxwell's Law of Reciprocal Deflections

- Now consider if the load at A were applied first and alone. The structure would deflect and the load would do work as it caused the deflection.
- Then apply the load at B. The load at A would deflect, thus it would do work.
- The load at B would do work as it caused the deflection.

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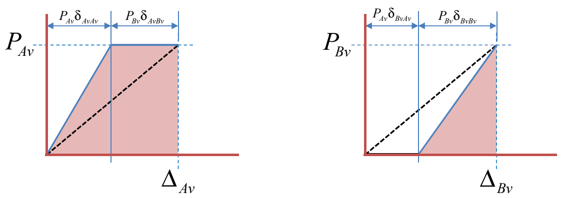
Maxwell's Law of Reciprocal Deflections

Work done when loads are applied independently



$$W_e = \frac{P_{Av}}{2}\left(P_{Av}\delta_{AvAv}\right) + P_{Av}\left(P_{Bv}\delta_{AvBv}\right) + \frac{P_{Bv}}{2}\left(P_{Bv}\delta_{BvBv}\right)$$

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Maxwell's Law of Reciprocal Deflections

The sequence of loading, however, will not affect the total external work done. Therefore, setting these equations equal yields:

$$\frac{P_{Av}}{2}\left(P_{Av}\delta_{AvAv} + P_{Bv}\delta_{AvBv}\right) + \frac{P_{Bv}}{2}\left(P_{Av}\delta_{BvAv} + P_{Bv}\delta_{BvBv}\right) = \frac{P_{Av}}{2}\left(P_{Av}\delta_{AvAv}\right) + P_{Av}\left(P_{Bv}\delta_{AvBv}\right) + \frac{P_{Bv}}{2}\left(P_{Bv}\delta_{BvBv}\right)$$

$$\frac{P_{Av}}{2}\left(P_{Bv}\delta_{AvBv}\right) + \frac{P_{Bv}}{2}\left(P_{Av}\delta_{BvAv}\right) = P_{Av}\left(P_{Bv}\delta_{AvBv}\right)$$

$$\delta_{BvAv} = \delta_{AvBv}$$

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Maxwell's Law of Reciprocal Deflections

- Proposition 1: *Any linear deflection component of any point A, resulting from the application of a unit force at any other point B, is equal in magnitude to the linear deflection component of B (in the direction of the first applied force at B) resulting from the application of a unit force at A applied in the direction of the original deflection component of A.*

$$\delta_{BvAv} = \delta_{AvBv}$$

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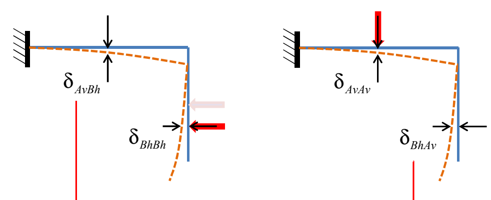
Maxwell's Law of Reciprocal Deflections

- A more general selection of deflections, rotations, forces and moments will result in a more complete understanding of the law.



$$\delta_{AvBh} = \delta_{BhAv}$$

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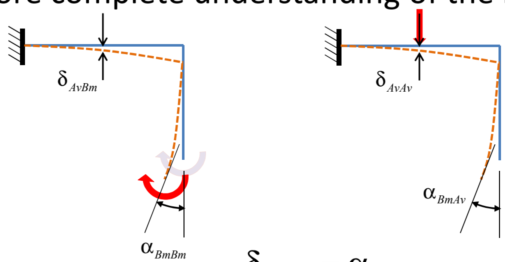
Maxwell's Law of Reciprocal Deflections

- A more general selection of deflections, rotations, forces and moments will result in a more complete understanding of the law.



$$\delta_{AvBm} = \alpha_{BmAv}$$

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Maxwell's Law of Reciprocal Deflections

- The great value of Maxwell's Law of Reciprocal Deflections will not be apparent until its use results in considerable savings of analysis effort later in our course.

$$\delta_{AvBv} = \delta_{BvAv}$$
$$\delta_{AvBh} = \delta_{BhAv}$$
$$\delta_{AvBm} = \alpha_{BmAv}$$

Think about this, how can a deflection be equal to a rotation?

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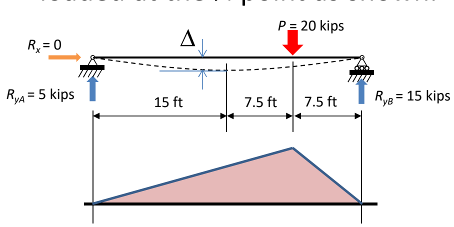
Deflections for Other Loads

Determine the deflection at mid-span for the beam loaded at the ¼ point as shown.



The internal work, or strain energy, equations can be developed just as before.

However, the external work equation will not include the desired deflection, Δ.

If we wanted the deflection under the load we would be OK, but that is not what is required.

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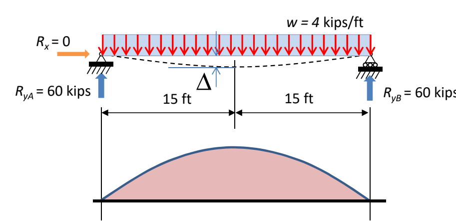
Deflections for Other Loads

Determine the deflection at mid-span for the beam loaded with a uniform load as shown.



As for the previous beam, the internal work equations can be developed easily.

However, the external work equation must include loads at points other than that of the desired deflection, Δ.

So again we have a problem.

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Virtual Work

- The principle of virtual work is based on the principle of virtual velocities presented by Johann Bernoulli in 1717.
- It is the most versatile method available for evaluating elastic deflections of structures.
- As was the case for the work solution we addressed earlier, it will be capable of accounting for all forms of internal strain energy.

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Virtual Work

- Principle of Virtual Work: *If a deformable structure, in equilibrium and sustaining a given load or system of loads, is subjected to a virtual deformation as the result of some additional action, the external virtual work of the given load or system of loads is equal to the internal virtual work of the stresses caused by the given load or system of loads.*

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Summary

- Developed an understanding of work and used it with the principle of virtual displacements to determine reactions.
- Derived equations for strain energy due to axial, flexural, shear and torsion forces and moments.
- Used real work to determine deflections for example beams.
- Established a systematic approach to notation for deflections.
- Illustrated the limitations of real work for calculating all the deflections that might be of interest.

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Lesson 3

- Deflections by Virtual Work
  - Develop the method of virtual work
  - Calculate deflections due to axial forces
  - Investigate schemes for writing moment equations
  - Determine deflections and rotations for beams, frames and trusses.
  - Investigate deflections due to temperature changes

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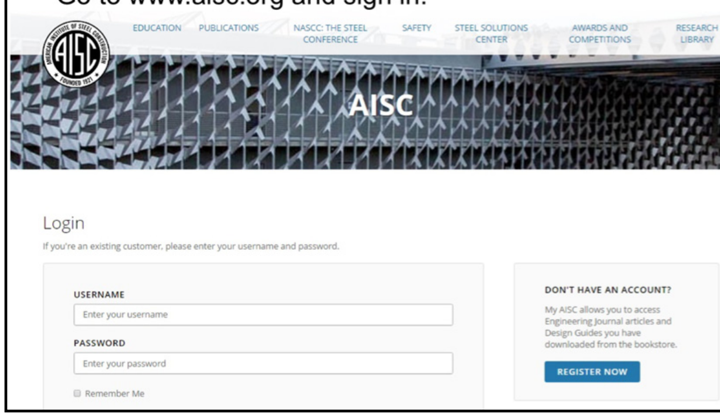

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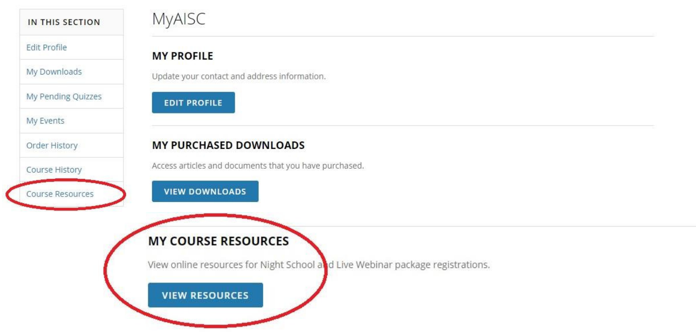

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