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Design of Curved Members

Part 1: Design of Vertically-Curved Members

December 5, 2019



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AISC Live Webinars

Course Description

Design of Vertically-Curved Members
December 5, 2019

While curved members can be both structurally efficient and aesthetically interesting, their behavior can be much different than their straight counterparts. This two-part webinar series, presented by the author of AISC Design Guide 33 – Curved Member Design, provides guidance on how to use the AISC Specification to design curved members. This 'equivalent straight member' approach allows the use of existing commercial software for curved member design by modifying effective length factors and lateral-torsional buckling modification factors to account for the curvature. This course will provide a brief overview of the design guide and detailed design information on both vertically- and horizontally-curved members.

This session will review the design of vertically-curved members for axial, flexural, and combined loading. A special buckling limit state, unique to vertically-curved members, will be introduced. Finally, the concepts will be illustrated with a design example.



AISC Live Webinars

Learning Objectives

- Identify the phenomenon of snap-through buckling, and how it can be avoided through arch geometry.
- Describe how in-plane flexural buckling is influenced by end conditions and the geometrical form of an arch.
- Explain how to calculate effective length factors that allow one to design vertically-curved members for axial forces buckling out-of-plane, using code provisions written for straight members.
- Explain how to calculate modification factors that allow one to design vertically-curved members for lateral-torsional buckling, using code provisions written for straight members.



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Design of Curved Members

Part 1: Design of Vertically-Curved Members
December 5, 2019



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Course Outline

- Session 1 (December 5)
 - Introduction
 - Overview of Design Guide 33
 - Design of vertically-curved members
- Session 2 (December 12)
 - Design of horizontally-curved members



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General Information on Curved Members

Introduction



Introduction

Vertically-Curved Members



Photographs courtesy of the AISC Bender/Roller Committee



Introduction

Horizontally-Curved Members



Photographs courtesy of the AISC Bender/Roller Committee



General Information on Curved Members

Overview of Design Guide 33



Design Guide Overview

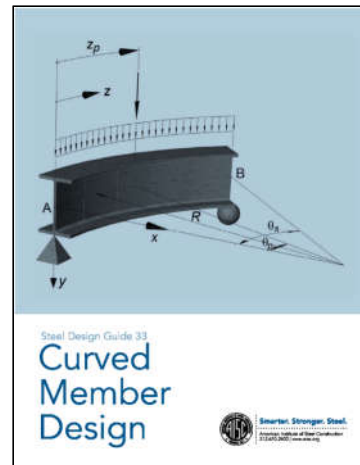
Purpose of Design Guide 33

- Design guidance
 - Vertically-curved members
 - Horizontally-curved members
 - Connections



Design Guide Overview

- Practical information
 - Fabrication
 - Detailing



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Design Guide Overview

Contents of Design Guide 33

- Chapter 1: Introduction
 - Typical applications of curved members
- Chapter 2: Curving Steel Members
 - Bending geometries and processes



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Design Guide Overview

Contents of Design Guide 33

- Chapter 3: Design for Bending
 - Preventing fracture and distortion during the bending operation
- Chapter 4: Fabrication and Detailing
 - Tolerances, fabrication and detailing



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Design Guide Overview

- Chapter 5: General Design Issues
 - Material properties, contract documents, etc.
- Chapter 6: Vertically-Curved Members
- Chapter 7: Horizontally-Curved Members



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Design Guide Overview

- Chapter 8: Design Examples
- Glossary
- List of bender-roller companies



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Vertically-Curved Members

Session Description



Session Description

- Axial strength
- Flexural strength
- Combined loads
- Design Example



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Vertically-Curved Members

Axial Strength



Axial Strength

Arches

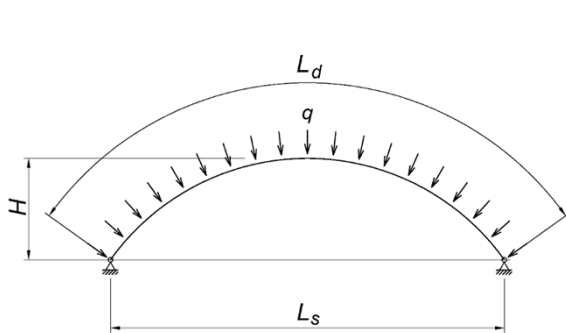
- Structurally efficient
- Loads carried primarily by compression



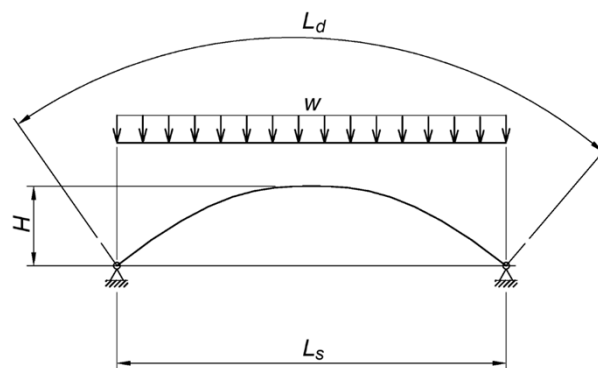
Photograph courtesy of the AISC Bender/Roller Committee



Axial Strength



Circular



Parabolic



Vertically-Curved Members

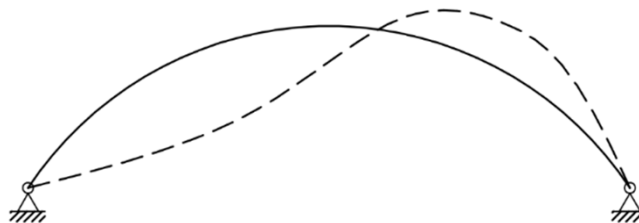
Axial Strength

In-Plane Strength



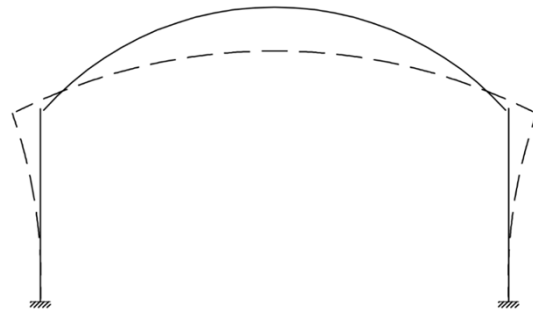
In-Plane Strength

- Curved Members \neq Straight Members
- Buckled Shape



In-Plane Strength

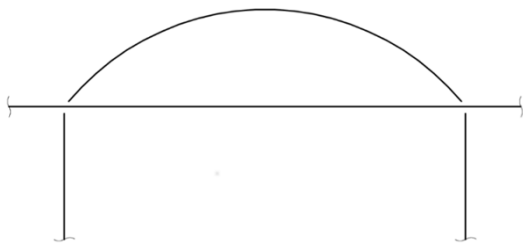
- Support spreading
 - Increases deflection
 - Can lead to collapse



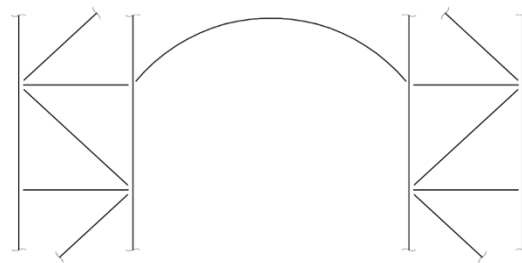
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In-Plane Strength

- Horizontal restraints



Tension Tie



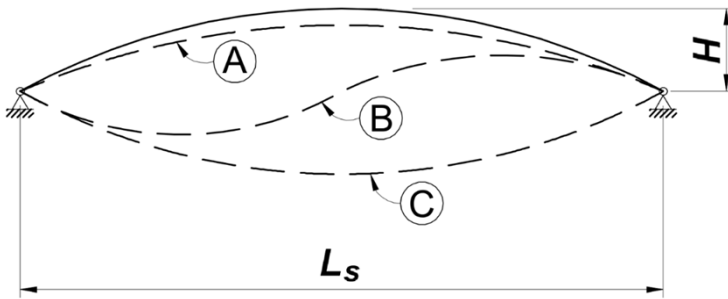
Vertical Truss



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In-Plane Strength

Snap-Through Buckling



- A: Deflected shape
- B: Antisymmetric mode
- C: Symmetric mode



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In-Plane Strength

Snap-Through Buckling

- Sensitive to second-order effects
- Sensitive to support spreading
- Difficult to predict

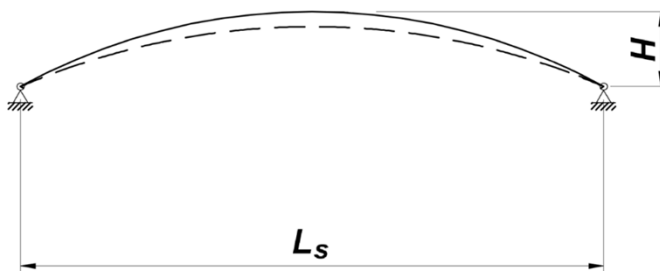


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In-Plane Strength

Snap-Through Buckling

- Generally, not critical for arches with:
 - Rigid supports
 - $H/L_s > 0.2$

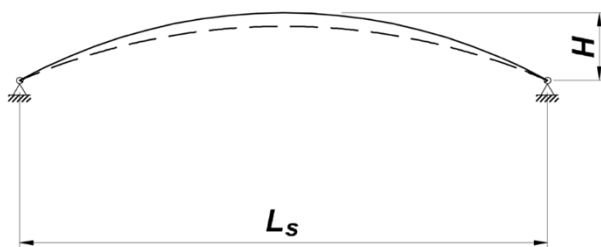


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In-Plane Strength

Snap-Through Buckling

- When $H/L_s \leq 0.2$, L_s/r_i must exceed $(L_s/r_i)_{crit}$



r_i = in-plane radius of gyration



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In-Plane Strength

		$(L_s/r_i)_{crit}$		
		H/L_s		
		0.10	0.15	0.20
End Conditions	Pinned	59	36	35
	Fixed	150	71	68



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In-Plane Strength

- Design as a straight column
- Flexural buckling provisions in AISC *Specification* Section E3

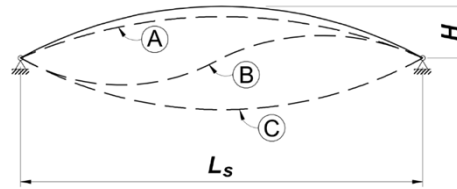
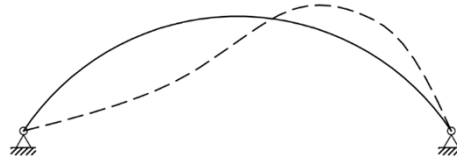


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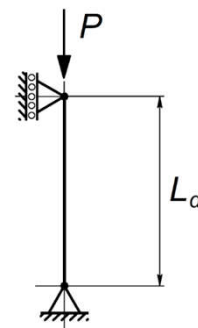
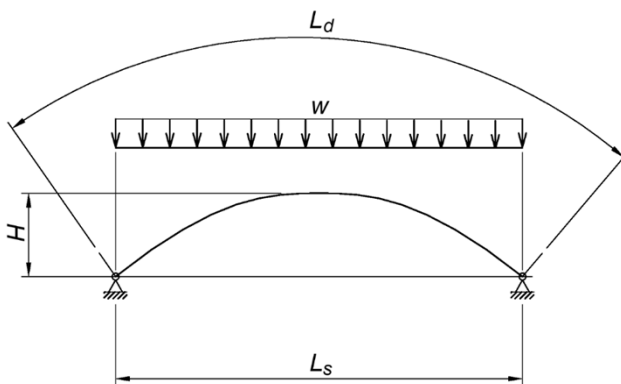
In-Plane Strength

- Applicable only for flexural buckling
- Not valid for snap-through buckling



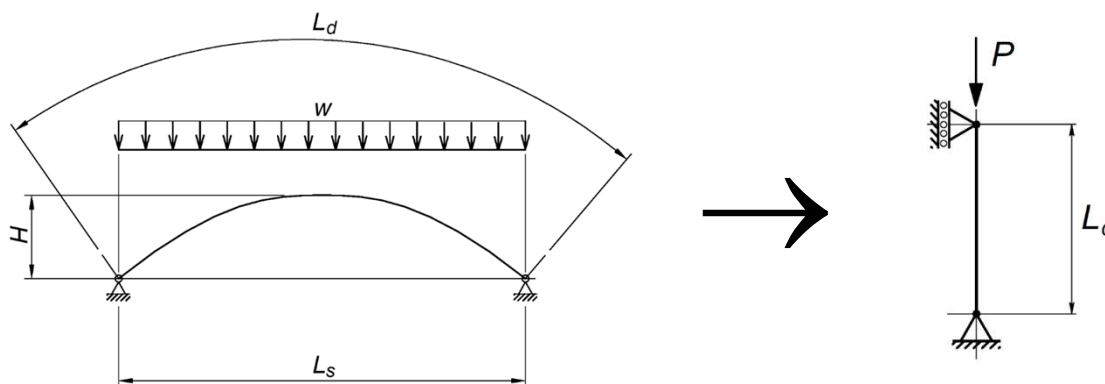
In-Plane Strength

- Unbraced length, $L \rightarrow L_d$



In-Plane Strength

- P = maximum axial load in the arch



In-Plane Strength

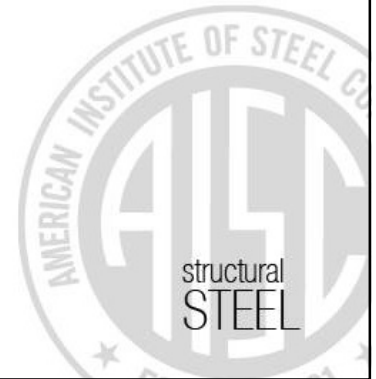
In-plane effective length factor, K_i			
Arch Form	End Conditions	H/L_s	K_i
Circular	Pinned	$0.1 \leq H/L_s \leq 0.3$	0.55
		$0.3 \leq H/L_s \leq 0.5$	0.60
	Fixed	All	0.40
Parabolic	Pinned	All	0.50
	Fixed	$0.1 \leq H/L_s < 0.3$	0.40
		$0.3 \leq H/L_s \leq 1.0$	0.35



Vertically-Curved Members

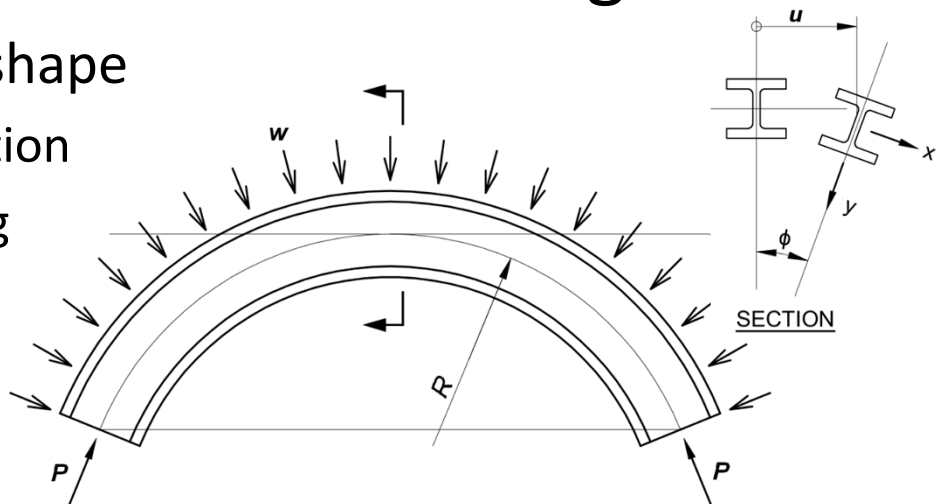
Axial Strength

Out-of-Plane Strength



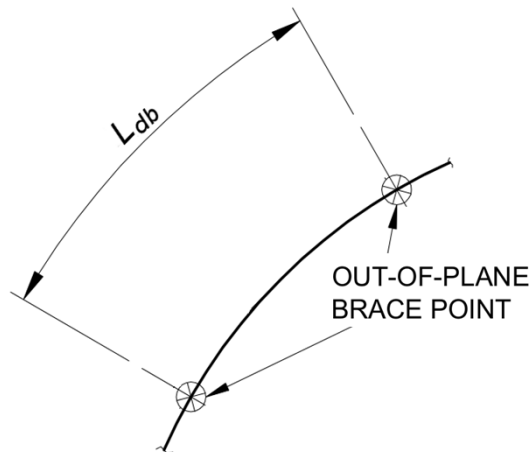
Out-of-Plane Strength

- Buckled shape
 - Translation
 - Twisting



Out-of-Plane Strength

- Most arches are braced against out-of-plane translation
- Each segment can buckle between brace points



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Out-of-Plane Strength

- Design as a straight column
- Flexural buckling provisions in AISC *Specification* Section E3

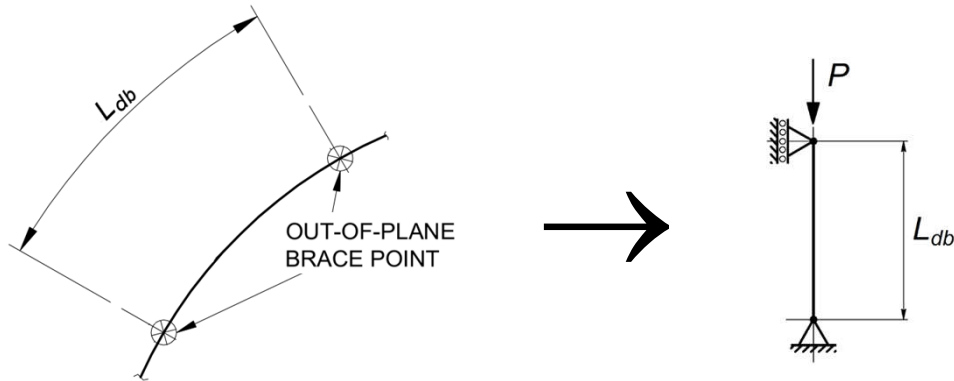


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Out-of-Plane Strength

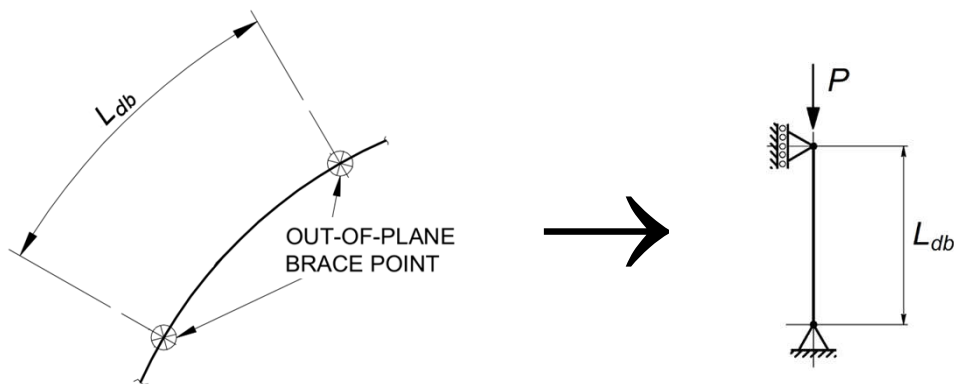
- Unbraced length, $L \rightarrow L_{db}$



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Out-of-Plane Strength

- P = maximum load in the segment

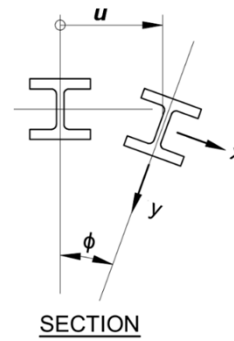
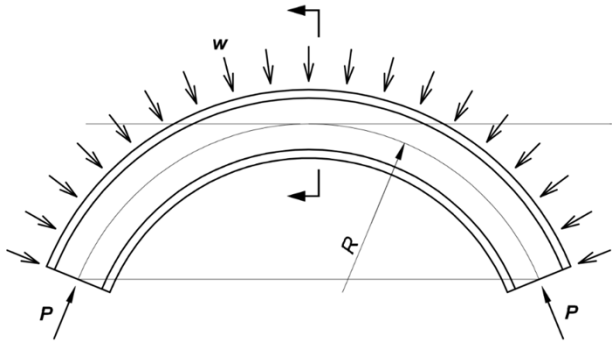


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Out-of-Plane Strength

- Flexural-torsional buckling → flexural buckling



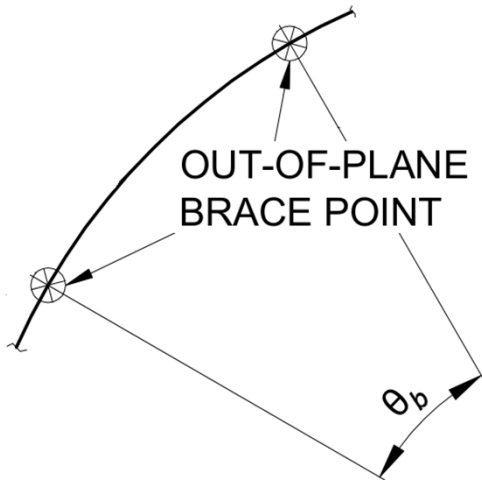
Out-of-Plane Strength

- Effective length factor, K_o : Circular doubly-symmetric segments

$$K_o = \frac{\sqrt{1 + \frac{1}{C_o} \left(\frac{\theta_b}{\pi} \right)^2}}{1 - \left(\frac{\theta_b}{\pi} \right)^2}$$



Out-of-Plane Strength



θ_b = angle between braces, rad



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Out-of-Plane Strength

$$C_o = \frac{1}{I_o} \left[\frac{GJ}{E} + C_w \left(\frac{\pi}{L_{db}} \right)^2 \right]$$

C_w = warping constant

J = torsional constant

I_o = moment of inertia \perp to the axis of curvature

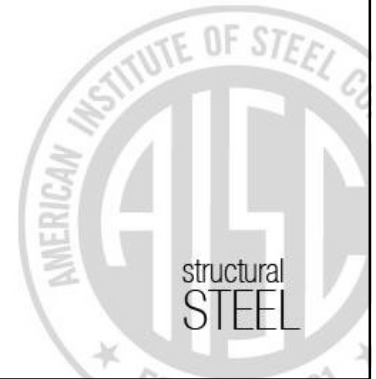


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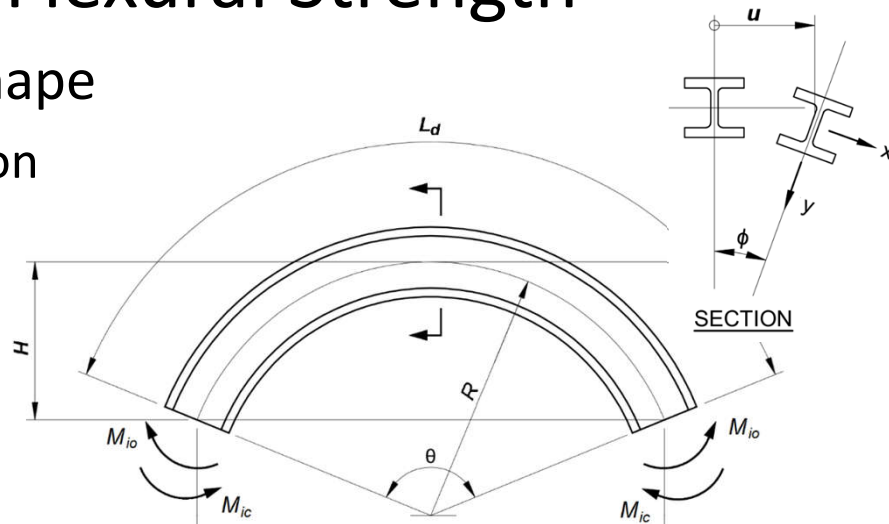
Vertically-Curved Members

Flexural Strength



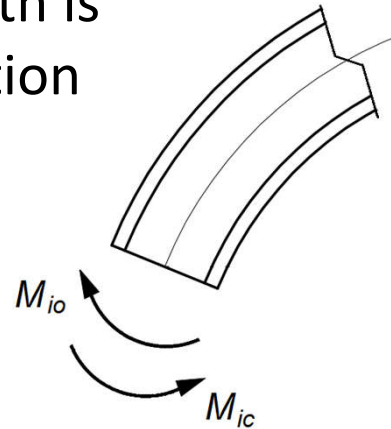
Flexural Strength

- Buckled shape
 - Translation
 - Twisting



Flexural Strength

- Lateral-torsional buckling strength is dependent on the loading direction
 - Opening moment, M_{io}
 - Closing moment, M_{ic}



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Flexural Strength

- Design as a straight beam
- Each unbraced segment is treated independently
- AISC *Specification* Chapter F
- Lateral-torsional buckling modification factor, $C_b \rightarrow C_{bi}$



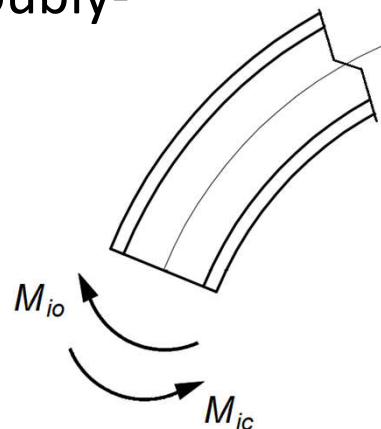
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Flexural Strength

Modification factor, C_{bi} : Circular doubly-symmetric segments

$$C_{bi} = C_{bs} \left(\sqrt{1 + C_a^2 - \frac{C_y C_z}{R^2 M_{es}^2}} \pm C_a \right)$$

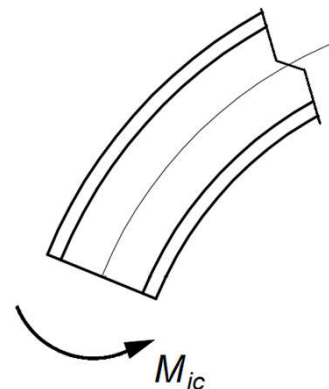


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Flexural Strength

Closing moments → positive root

$$C_{bi} = C_{bs} \left(\sqrt{1 + C_a^2 - \frac{C_y C_z}{R^2 M_{es}^2}} + C_a \right)$$



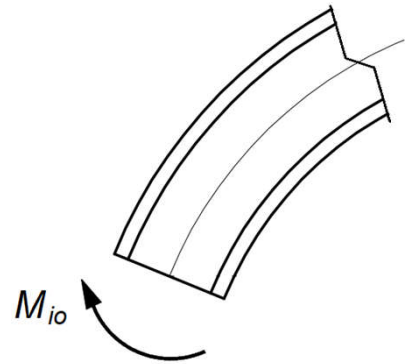
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Flexural Strength

Opening moments → negative root

$$C_{bi} = C_{bs} \left(\sqrt{1 + C_a^2 - \frac{C_y C_z}{R^2 M_{es}^2}} - C_a \right)$$



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Flexural Strength

$C_{bs} = C_b$ for an equivalent straight member (AISC *Specification* Equation F1-1)

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

M_{max} = absolute value of max. moment in the segment

$M_{A,B,C}$ = absolute value of moment at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ points



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Flexural Strength

M_{es} = elastic lateral-torsional buckling moment of the equivalent straight member subjected to uniform moment with a length equal to L_{db}

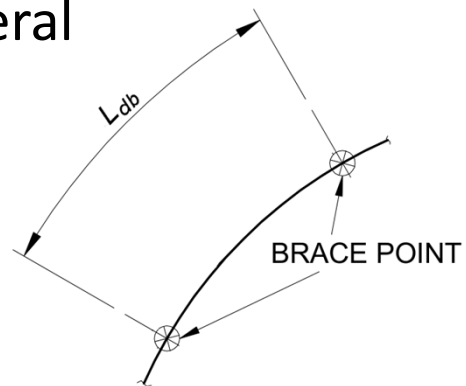
$$M_{es} = \frac{\pi}{L_{db}} \sqrt{EI_o GJ + \left(\frac{\pi E}{L_{db}}\right)^2 I_o C_w}$$



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Flexural Strength

L_{db} = developed length between points that are either braced against lateral displacement of the compression flange or twisting of the cross section



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Flexural Strength

$$C_a = \frac{C_y + C_z}{2RM_{es}}$$

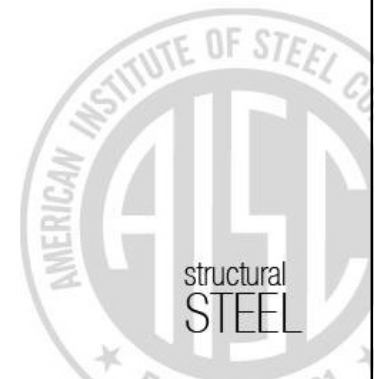
$$C_y = EI_o$$

$$C_z = GJ + \frac{\pi^2 EC_w}{L_{db}^2}$$



Vertically-Curved Members

Combined Loads



Combined Loads

Axial-Flexure Interaction

- AISC *Specification* Section H1 for doubly-symmetric members



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Combined Loads

Axial-Flexure Interaction

- When $P_r/P_c \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9} \left(\frac{M_r}{M_c} \right) \leq 1.0$$

$P_r M_r$ = required strengths $P_c M_c$ = available strengths



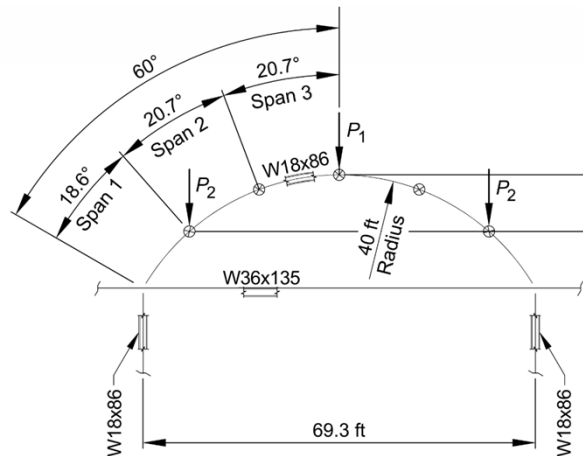
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Combined Loads

Axial-Flexure Interaction

- Strength must be verified at each segment
- Use the minimum P_r/P_c ratio for each segment



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Combined Loads

Axial-Flexure Interaction

- In-plane buckling: P_r/P_c is constant for all segments
- Out-of-plane buckling: both P_r and P_c can vary between segments



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Combined Loads

Second-Order Effects

- Amplified first-order moment: $M_{ri} = B_i M_{i1}$
- Second-order analysis (Section 6.3.1 of Design Guide 33)

M_{i1} = first-order moment about the axis of curvature



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Combined Loads

Second-Order Effects

$$B_i = \frac{1}{1 - \alpha \frac{P_r}{P_{ei}}} \quad \begin{array}{l} \alpha = 1.00 \text{ (LRFD)} \\ \alpha = 1.60 \text{ (ASD)} \end{array}$$

P_r = maximum axial load in the arch

P_{ei} = elastic critical load for in-plane buckling



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Combined Loads

Second-Order Effects

$$P_{ei} = \frac{\pi^2 E I_i}{(K_i L_d)^2}$$

I_i = moment of inertia about the axis of curvature

K_i = effective length factor for in-plane buckling

L_d = developed length of the arch



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Vertically-Curved Members

Design Example



Problem Statement

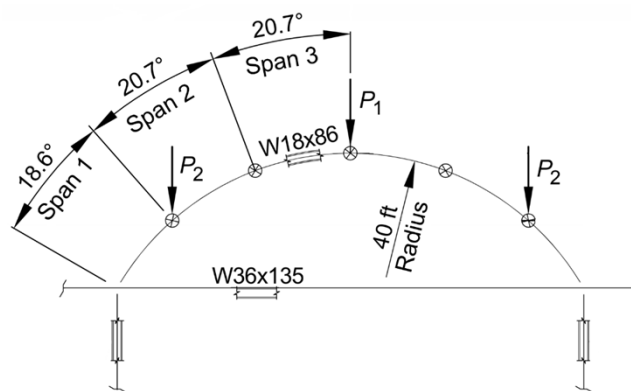
- Verify that the arch is adequate for the imposed loading
- Use LRFD



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Problem Statement

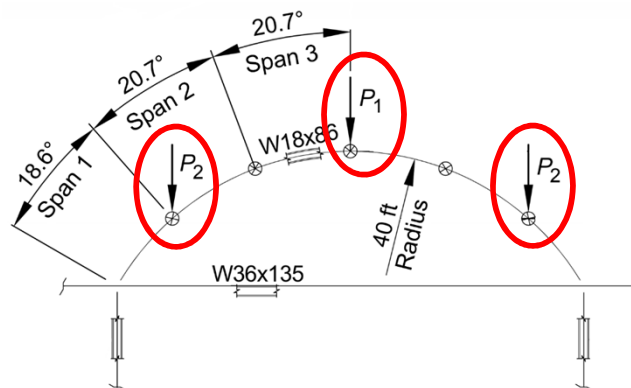
- Curved member
 - W18×86
 - ASTM A992
 - Bent the hard way
 - Circular curve



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Problem Statement

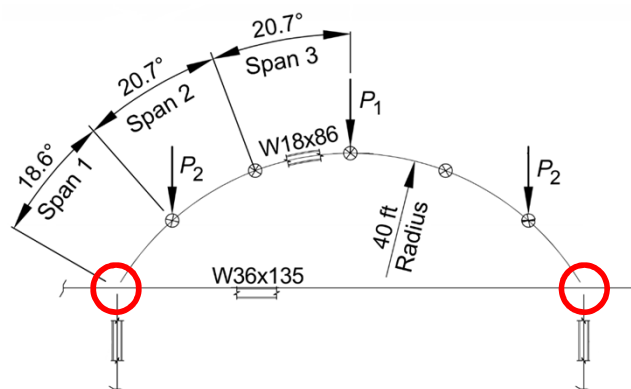
- The factored (LRFD) loads are
 - $P_{1u} = 120$ kips
 - $P_{2u} = 75$ kips



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Problem Statement

- Supports
 - Translation fixed in all directions
 - Rotation free in all directions

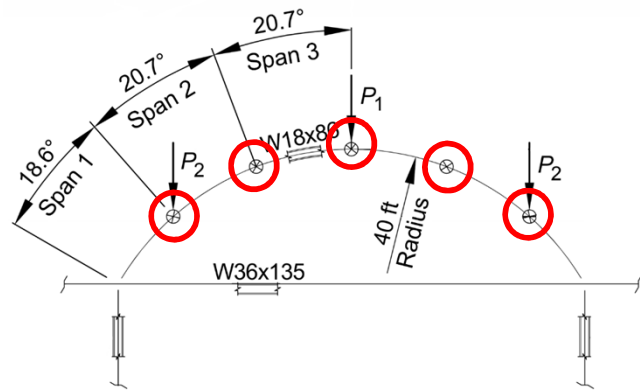


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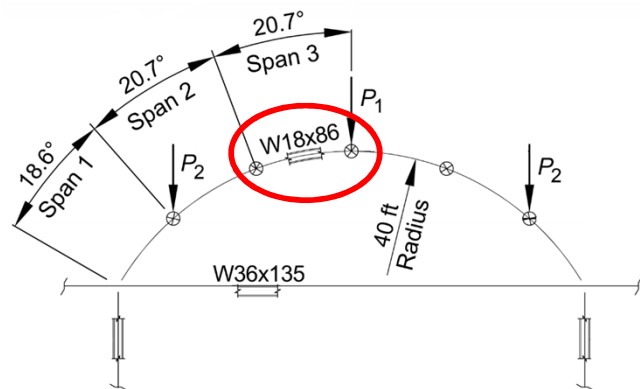
Problem Statement

- Braces
 - Prevent out-of-plane translation
 - Prevent torsional rotation



Problem Statement

- Assume Span 3 is critical



Vertically-Curved Members

Design Example

Properties



Properties

- Material properties of ASTM A992
(*AISC Manual* Table 2-4)

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$



Properties

- Dimensions of W18×86
(AISC *Manual* Table 1-1)

$$\begin{array}{ll} d = 18.4 \text{ in.} & t_w = 0.480 \text{ in.} \\ b_f = 11.1 \text{ in.} & t_f = 0.770 \text{ in.} \\ h_o = 17.6 \text{ in.} & \end{array}$$



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Properties

- Section properties of W18×86
(AISC *Manual* Table 1-1)

$$\begin{array}{ll} I_x = 1,530 \text{ in.}^4 & S_x = 166 \text{ in.}^3 \\ r_x = 7.77 \text{ in.} & Z_x = 186 \text{ in.}^3 \\ I_y = 175 \text{ in.}^4 & r_y = 2.63 \text{ in.} \\ J = 4.10 \text{ in.}^4 & C_w = 13,600 \text{ in.}^6 \\ r_{ts} = 3.05 \text{ in.} & \end{array}$$



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Vertically-Curved Members

Design Example

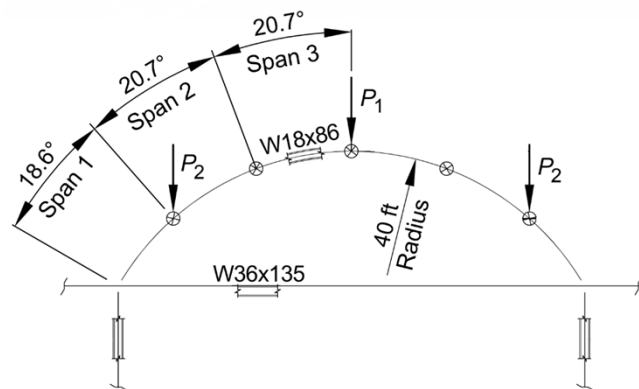
Arch Geometry



Arch Geometry

- Centroidal radius

$$R = (40 \text{ ft})(12 \text{ in./ft}) \\ = 480 \text{ in.}$$

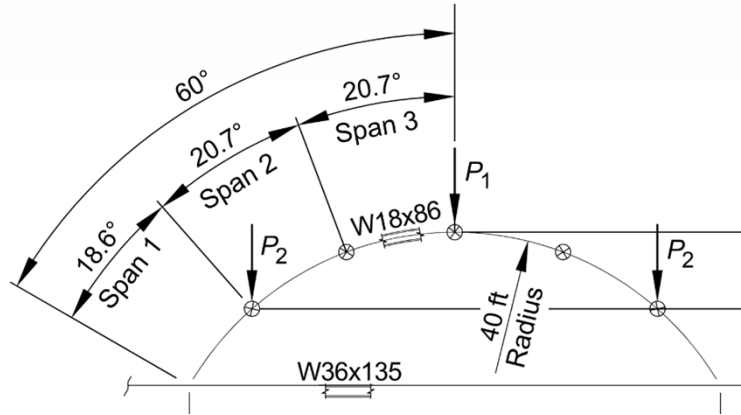


Arch Geometry

- Arch angle

$$\theta = (120^\circ) \left(\frac{\pi \text{ rad}}{180^\circ} \right)$$

$$= (2\pi / 3) \text{ rad}$$



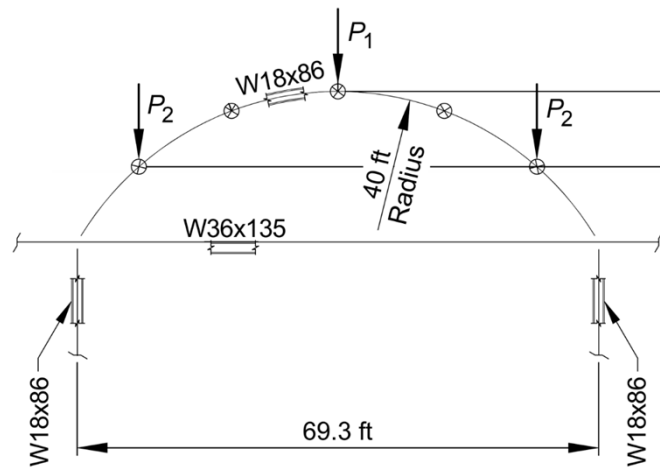
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Arch Geometry

- Span length (chord)

$$L_s = (69.3 \text{ ft})(12 \text{ in./ft})$$

$$= 832 \text{ in.}$$



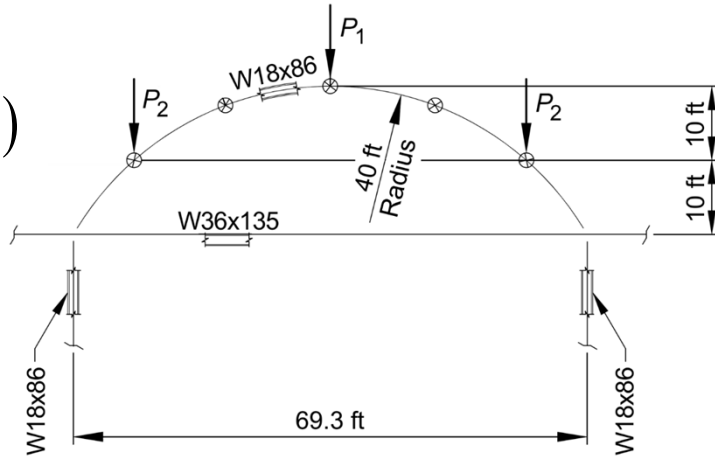
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Arch Geometry

- Rise

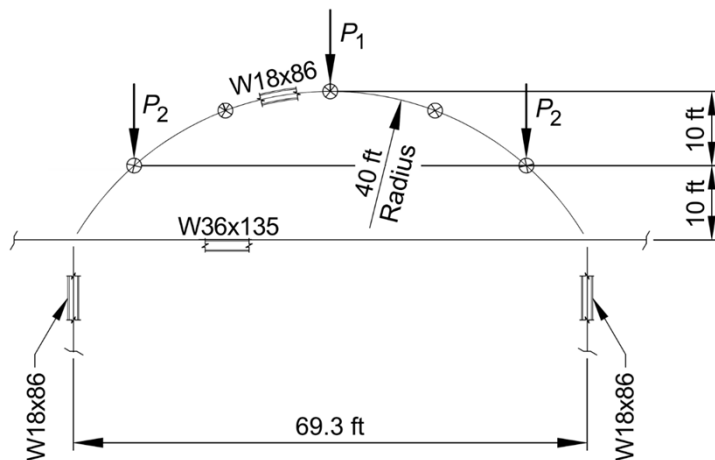
$$H = (20 \text{ ft})(12 \text{ in./ft}) = 240 \text{ in.}$$



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Arch Geometry

$$\frac{H}{L_s} = \frac{240 \text{ in.}}{832 \text{ in.}} = 0.288$$



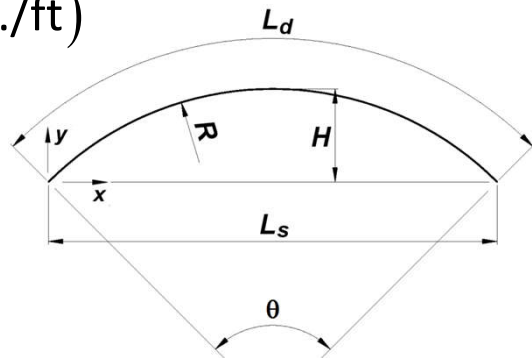
86



Arch Geometry

- Developed arc length

$$L_d = (40 \text{ ft}) \left[(2\pi / 3) \text{ rad} \right] (12 \text{ in./ft})$$
$$= 1,010 \text{ in.}$$



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Vertically-Curved Members

Design Example

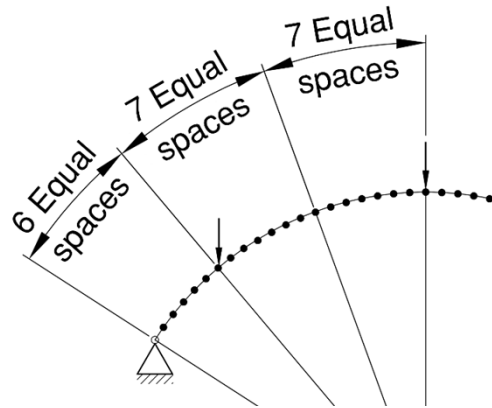
Structural Analysis



Structural Analysis

Finite Element Model

- Segmented spans
 - Straight beam elements
 - $\approx 3^\circ$ arc between nodes
- First-order analysis



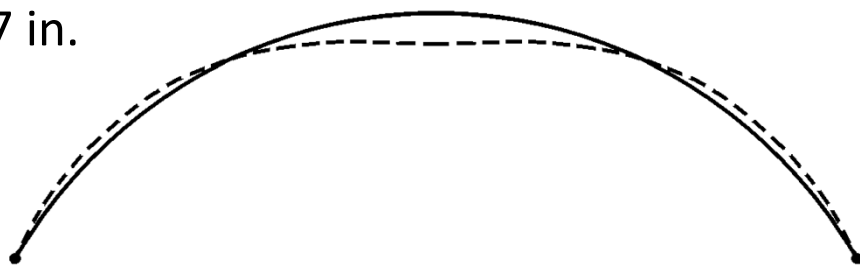
89

Structural Analysis

Deflection

- Maximum @ apex

$$\Delta_1 = 1.07 \text{ in.}$$



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Structural Analysis

Deflection

- Design Guide Section 6.3.1:

If Δ_1 is less than $H/40$ (using factored loads for LRFD or 1.6 times the service loads for ASD), a first-order finite element analysis is sufficiently accurate



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Structural Analysis

Deflection

$$\frac{H}{40} = \frac{240 \text{ in.}}{40} = 6.00 \text{ in.}$$

1.07 in. < 6.00 in. **o.k.**

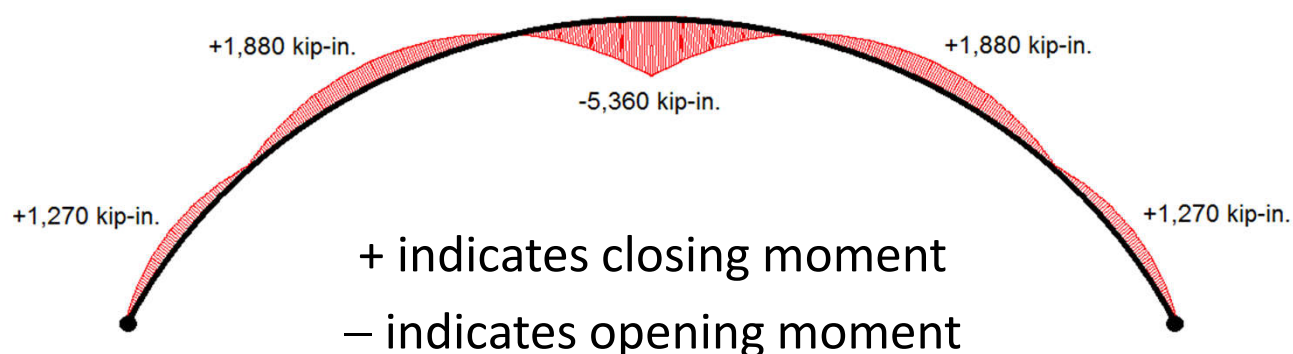


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Arch Geometry

In-Plane Moment



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Structural Analysis

Member Loads (kip, in.)			
Location	Axial	Moment	Shear
	P_u	M_{ux}	V_u
Supports	182	0	25.8
Apex	118	-5,360	57.5



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Structural Analysis

1 st -Order Member Loads (kip, in.)			
Location	Axial	Moment	Shear
	P_u	M_{ux}	V_u
Max./Min. @ Span 3	131	+1,380 -5,360	57.5



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Vertically-Curved Members

Design Example

Local Buckling



Local Buckling

- Calculations are the same as for a straight member
- Axial: $\lambda_f < \lambda_{rf}$ and $\lambda_w < \lambda_{rw}$ → the W18×86 is non-slender
- Flexure: $\lambda_f < \lambda_{pf}$ and $\lambda_w < \lambda_{pw}$ → the W18×86 is compact



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Vertically-Curved Members

Design Example

Shear Strength



Shear Strength

- AISC *Manual* Table 6-2

$$\phi_v V_n = 265 \text{ kips} > V_u = 57.5 \text{ kips} \quad \mathbf{o.k.}$$



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Vertically-Curved Members

Design Example

In-Plane Strength



In-Plane Strength

- Snap-through buckling is not critical because:
 - The supports are rigid
 - $H/L_s = 0.288 > 0.2$



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In-Plane Strength

- AISC *Specification* Section E3
- Radius of gyration about the axis of curvature:
 $r_i = r_x = 7.77$ in.

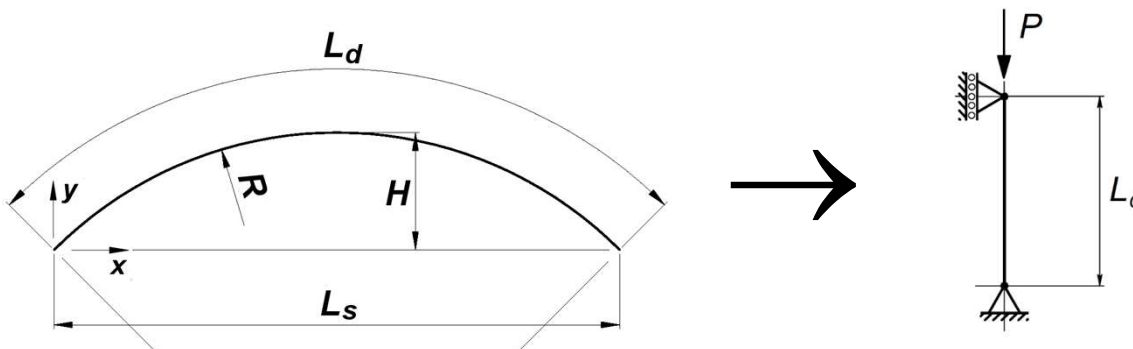


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In-Plane Strength

- Unbraced length: $L_d = 1,010$ in.



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In-Plane Strength

- For a circular arch with pinned end conditions and $H/L_s = 0.288$, $K_i = 0.55$

In-plane effective length factor, K_i			
Arch Form	End Conditions	H/L_s	K_i
Circular	Pinned	$0.1 \leq H/L_s \leq 0.3$	0.55
		$0.3 \leq H/L_s \leq 0.5$	0.60
	Fixed	All	0.40



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In-Plane Strength

$$\begin{aligned}\frac{L_c}{r} &= \frac{K_i L_d}{r_i} \\ &= \frac{(0.55)(1,010 \text{ in.})}{7.77 \text{ in.}} \\ &= 71.5\end{aligned}$$

AISC Manual Table 4-14: $\phi_c F_{cr} = 31.0 \text{ ksi}$



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In-Plane Strength

- The available strength is

$$\begin{aligned}\phi_c P_{ni} &= (31.0 \text{ ksi})(25.3 \text{ in.}^2) \\ &= 784 \text{ kips}\end{aligned}$$



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Vertically-Curved Members

Design Example

Out-of-Plane Strength



Out-of-Plane Strength

- *AISC Specification* Section E3
- Moment of inertia perpendicular to the axis of curvature: $I_o = I_y = 175 \text{ in.}^4$
- Radius of gyration perpendicular to the axis of curvature: $r_o = r_y = 2.63 \text{ in.}$

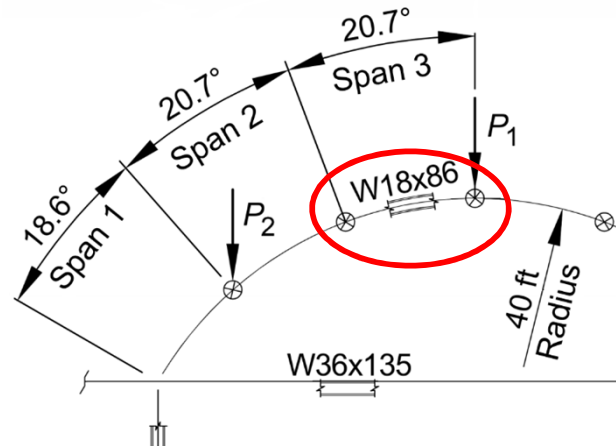


Out-of-Plane Strength

Span 3

$$\theta_b = (20.7^\circ) \left(\frac{\pi \text{ rad}}{180^\circ} \right)$$

$$= 0.361 \text{ rad}$$



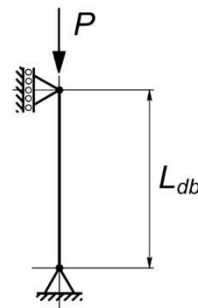
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Out-of-Plane Strength

$$L_{db} = R\theta_b$$

$$= (480 \text{ in.})(0.361 \text{ rad})$$

$$= 173 \text{ in.}$$



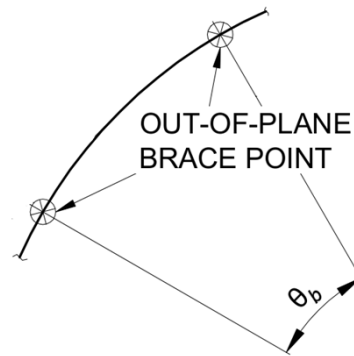
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Out-of-Plane Strength

- Effective length factor, K_o : Circular doubly-symmetric segments

$$K_o = \frac{\sqrt{1 + \frac{1}{C_o} \left(\frac{\theta_b}{\pi} \right)^2}}{1 - \left(\frac{\theta_b}{\pi} \right)^2}$$



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Out-of-Plane Strength

$$C_o = \frac{1}{I_o} \left[\frac{GJ}{E} + C_w \left(\frac{\pi}{L_{db}} \right)^2 \right]$$

$$= \frac{1}{175 \text{ in.}^4} \left[\frac{(11,200 \text{ ksi})(4.10 \text{ in.}^4)}{29,000 \text{ ksi}} + (13,600 \text{ in.}^6) \left(\frac{\pi}{173 \text{ in.}} \right)^2 \right]$$

$$= 0.0347$$



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Out-of-Plane Strength

$$K_o = \frac{\sqrt{1 + \frac{1}{C_o} \left(\frac{\theta_b}{\pi} \right)^2}}{1 - \left(\frac{\theta_b}{\pi} \right)^2} = \frac{\sqrt{1 + \frac{1}{0.0347} \left(\frac{0.361 \text{ rad}}{\pi} \right)^2}}{1 - \left(\frac{0.361 \text{ rad}}{\pi} \right)^2} = 1.19$$



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Out-of-Plane Strength

$$\begin{aligned} \frac{L_c}{r} &= \frac{K_o L_{db}}{r_o} \\ &= \frac{(1.19)(173 \text{ in.})}{2.63 \text{ in.}} \\ &= 78.3 \end{aligned}$$

AISC Manual Table 4-14: $\phi_c F_{cr} = 28.7 \text{ ksi}$



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Out-of-Plane Strength

- The available strength is

$$\begin{aligned}\phi_c P_{no} &= (28.7 \text{ ksi})(25.3 \text{ in.}^2) \\ &= 726 \text{ kips}\end{aligned}$$



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Vertically-Curved Members

Design Example

Second-Order Effects



Second-Order Effects

- Amplified first-order analysis
- Second-order moment: $M_{ux2} = B_i M_{ux}$

$$B_i = \frac{1}{1 - \alpha \frac{P_u}{P_{ei}}}$$

P_{ei} = elastic critical load for in-plane buckling

$\alpha = 1.00$ (LRFD)



REF: Session 2 Slide 87

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Second-Order Effects

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2}$$

$$= \frac{\pi^2 (29,000 \text{ ksi})}{(71.5)^2}$$

$$= 56.0 \text{ ksi}$$

$$P_{ei} = F_e A_g$$

$$= (56.0 \text{ ksi})(25.3 \text{ in.}^2)$$

$$= 1,420 \text{ kips}$$



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Second-Order Effects

$$B_i = \frac{1}{1 - \alpha \frac{P_u}{P_{ei}}} = \frac{1}{1 - (1.0) \left(\frac{182 \text{ kips}}{1,420 \text{ kips}} \right)} = 1.15$$



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Second-Order Effects

2 nd -Order Moments (kip-in.)	
Location	M_{ux2}
Max./Min. @	+1,590
Span 3	-6,160



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Vertically-Curved Members

Design Example

Flexural Strength



Flexural Strength

- *AISC Specification* Section F2
- Moment of inertia perpendicular to the axis of curvature: $I_o = I_y = 175 \text{ in.}^4$



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Flexural Strength

$$\begin{aligned} M_p &= F_y Z_x \\ &= (50 \text{ ksi})(186 \text{ in.}^3) \\ &= 9,300 \text{ kip-in.} \end{aligned}$$

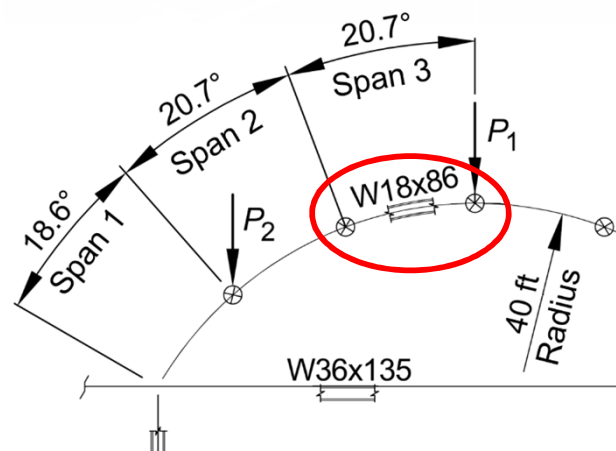


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Flexural Strength

Span 3

$$L_{db} = 173 \text{ in.}$$



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Flexural Strength

M_{es} = elastic lateral-torsional buckling moment of the equivalent straight member subjected to uniform moment with a length equal to L_{db}

$$M_{es} = \frac{\pi}{L_{db}} \sqrt{EI_o GJ + \left(\frac{\pi E}{L_{db}}\right)^2 I_o C_w} = 17,200 \text{ kip-in.}$$



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Flexural Strength

$$\begin{aligned} C_y &= EI_o \\ &= (29,000 \text{ ksi})(175 \text{ in.}^4) \\ &= 5,080,000 \text{ kip-in.}^2 \end{aligned}$$



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Flexural Strength

$$\begin{aligned} C_z &= GJ + \frac{\pi^2 EC_w}{L_{db}^2} \\ &= (11,200 \text{ ksi})(4.10 \text{ in.}^4) + \frac{\pi^2 (29,000 \text{ ksi})(13,600 \text{ in.}^6)}{(173 \text{ in.})^2} \\ &= 176,000 \text{ kip-in.}^2 \end{aligned}$$



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Flexural Strength

$$\begin{aligned} C_a &= \frac{C_y + C_z}{2RM_{es}} \\ &= \frac{5,080,000 \text{ kip-in.}^2 + 176,000 \text{ kip-in.}^2}{(2)(480 \text{ in.})(17,200 \text{ kip-in.})} \\ &= 0.318 \end{aligned}$$

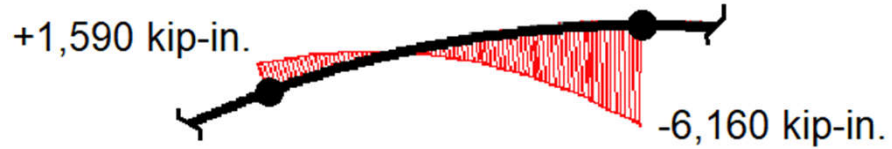


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Flexural Strength

- Span 3



$$C_{bs} = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} = 2.38$$



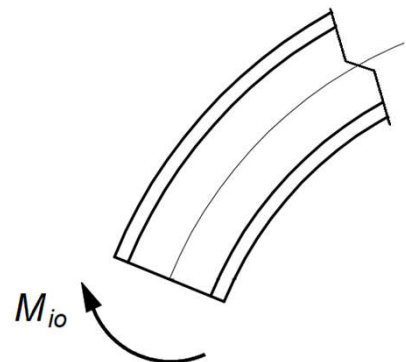
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Flexural Strength

Opening moments → negative root

$$C_{bi} = C_{bs} \left(\sqrt{1 + C_a^2 - \frac{C_y C_z}{R^2 M_{es}^2}} - C_a \right)$$

$$= 1.75$$



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Flexural Strength

- AISC *Specification* Section F2
 - $L_b = L_{db} = 173 \text{ in.} = 14.4 \text{ ft}$
 - $C_b = C_{bi} = 1.75$
- AISC *Manual* Table 3-6
 - $L_p = 9.29 \text{ ft}$
 - $L_r = 28.6 \text{ ft}$



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Flexural Strength

- $L_p < L_b < L_r \rightarrow$ Use AISC *Spec.* Eq. F2-2

$$M_n = C_b \left[M_p - (M_p - 0.7F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$
$$= 14,700 \text{ kip-in.} > 9,300 \text{ kip-in.}$$



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Flexural Strength

- The available strength is

$$\begin{aligned}\phi_b M_n &= 0.90(9,300 \text{ kip-in.}) \\ &= 8,370 \text{ kip-in.}\end{aligned}$$



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Vertically-Curved Members

Design Example

Combined Loading



Combined Loading

- AISC *Specification* Section H1
- In-plane buckling
- Out-of-plane buckling



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Combined Loading

In-Plane Buckling

- The largest flexural load ratio is at the apex
- The largest axial load ratio is at the supports

$$\frac{P_u}{\phi_c P_{ni}} = \frac{182 \text{ kips}}{784 \text{ kips}} = 0.232$$



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Combined Loading

0.232 > 0.2 → AISC *Specification* Equation H1-1a

$$\frac{P_u}{\phi_c P_{ni}} + \frac{8}{9} \left(\frac{M_u}{\phi_b M_n} \right) \leq 1.0$$

$$0.232 + \left(\frac{8}{9} \right) \left(\frac{6,160 \text{ kip-in.}}{8,370 \text{ kip-in.}} \right) \leq 1.0$$

0.886 < 1.0 **o.k.**



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Combined Loading

Out-of-Plane Buckling (Span 3)

- Largest flexural load in the span
- Largest axial load in the span

$$\frac{P_u}{\phi_c P_{no}} = \frac{131 \text{ kips}}{726 \text{ kips}} = 0.180$$



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Combined Loading

$0.180 < 0.2 \rightarrow$ AISC *Specification* Equation H1-1b

$$\frac{P_u}{2\phi_c P_{no}} + \frac{M_u}{\phi_b M_n} \leq 1.0$$
$$\frac{0.180}{2} + \frac{6,160 \text{ kip-in.}}{8,370 \text{ kip-in.}} \leq 1.0$$

$0.826 < 1.0$ **o.k.**



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Combined Loading

Design Summary

- The W18×86 is adequate
- See Design Guide Example 8.1 for further calculations
 - Local strength
 - Connections



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Conclusions

Vertically-Curved Members

- Design as an equivalent straight member
- Subjected to both axial and flexural loads
- Axial strength is calculated with K_i and K_o
- Flexural strength is calculated with C_{bi}



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Question time



AISC | Questions?



CEU / PDH Certificates

- You will receive an email on how to report attendance from:
registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!



CEU / PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
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